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Robust and fragile Werner states in the collective dephasing

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Abstract. We investigate the concurrence and Bell violation of the standard Werner state or Werner-like states in the presence of collective dephasing. It is shown that the standard Werner state and certain kinds of Werner-like states are robust against the collective dephasing, and some kinds of Werner-like states is fragile and becomes completely disentangled in a finite-time. The threshold time of complete disentanglement of the fragile Werner-like states is given. The influence of external driving field on the finite-time disentanglement of the standard Werner state or Werner-like states is discussed. Furthermore, we present a simple method to control the stationary state entanglement and Bell violation of two qubits. Finally, we show that the theoretical calculations of fidelity based on the initial Werner state assumption well agree with previous experimental results.

PACS. 03.65.Ud Entanglement and quantum nonlocality – 03.67.-a Quantum information – 05.40.Ca Noise

Quantum entanglement plays a crucial role in quantum information processes [1]. Entanglement can exhibit the nature of a nonlocal correlation between quantum systems that have no classical interpretation. However, real quantum systems will unavoidably be influenced by surrounding environments. The interaction between the environment and quantum systems of interest can lead to decoherence. Certain kind of the decoherence is the collective dephasing, which occurs in the physical systems such as trapped ions, quantum dots, or atoms inside a cavity. Collective dephasing allows the existence of the so-called decoherence-free subspace [2].

Recently, the Werner or Werner-like states [3–6] has intrigued many interests for the applications in quantum information processes. Lee and Kim have discussed the entanglement teleportation via the Werner states [7]. Hiroshima and Ishizaka have studied the entanglement of the so-called Werner derivative, which is the state transformed by nonlocal unitary-local or nonlocal-operations from a Werner state [8]. Miranowicz has examined the Bell violation and entanglement of Werner states of two qubits in independent decay channels [9]. The experimental preparation and characterization of the Werner states have also been reported. An experiment for preparing the Werner state via spontaneous parametric down-conversion has been put forward [10]. Altepeter et al. have experimentally produced the Werner state and applied it in the ancilla-assisted process tomography [11]. Barbieri et al. have presented a novel technique for generating and characterizing two-photon polarization Werner states [12], which is based on the peculiar spatial characteristics of a high brilliance source of entangled pairs.

The disentanglement of entangled states of qubits is a very important issue for quantum information processes, such as the solid state quantum computation. Yu and Eberly have found that the time for decay of the qubit entanglement can be significantly shorter than the time for local dephasing of the individual qubits [13,14]. In this paper, we investigate the entanglement and Bell violation of the standard Werner state or Werner-like states in the presence of collective dephasing. The entanglement quantified by the concurrence and Bell violation of the collective dephasing Werner-like state are analyzed. We find that the standard Werner state and certain kinds of Werner-like states are robust against the collective dephasing, and some kinds of Werner-like states are fragile and become completely disentangled in a finite-time. The threshold time for the complete disappearance of the entanglement of the fragile Werner-like states is obtained. We also provide an explicit example to clarify how the pure maximally entangled states of two qubits can become separable in the finite time due to the joint action of collective dephasing and the external driving fields.

Meanwhile, there have been several proposals for controlling the entanglement of the qubits in the presence of dephasing or dissipation, such as quantum error-correcting approach [15–17], quantum error-avoiding approach [18,19], and loop control strategies [20] etc. Here, we present a possible way to preserve the entanglement

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of two qubits initially in the fragile entangled state under the collective dephasing environment. It is shown that the external local driving field with an appropriate finite action time can effectively transform the fragile entangled state into a robust entangled state.

The standard two-qubit Werner state is defined by [3]

$$\rho_W = r|\Psi^-\rangle\langle\Psi^-| + \frac{1-r}{4} I \otimes I, \tag{1}$$

where $r \in [0,1]$, and $|\Psi^-\rangle$ is the singlet state of two qubits. I is the identity operator of a single qubit. Recently, definition (1) is generalized to include the following states of two qubits [4–6]

$$\rho_W' = r|M\rangle\langle M| + \frac{1-r}{4} I \otimes I, \tag{2}$$

where $|M\rangle$ are any two-qubit maximally entangled states. Both the Werner state (1) and the Werner-like state (2) are very important in quantum information. The Werner state (1) is highly symmetric and $SU(2) \otimes SU(2)$ invariant [3,21].

The collective dephasing can be described by the master equation [19]

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{\gamma}{2}(2\hat{J}_z\hat{\rho}\hat{J}_z - \hat{J}_z^2\hat{\rho} - \hat{\rho}\hat{J}_z^2), \tag{3}$$

where γ is the dephasing rate. $\hat{J}_z$ are the collective spin operator defined by

$$\hat{J}_z = \sum_{i=1}^{2} \hat{\sigma}_z^{(i)}/2, \tag{4}$$

where $\hat{\sigma}_z$ for each qubit is defined by $\hat{\sigma}_z = |1\rangle\langle 1| - |0\rangle\langle 0|$. Firstly, it is obvious that the standard Werner state (1) is completely decoupled from the collective dephasing. So in the presence of collective dephasing, the state (1) belongs to the decoherence-free subspace, and it maintain its entanglement invariant. Then, we want to know how the collective dephasing affects the Werner-like states defined by equation (2). For simplicity, we only consider three states defined by equation (2) in which the maximally entangled states are the Bell triglet states. If two qubits are initially in the Werner-like state

$$\rho_W' = r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4} I \otimes I, \tag{5}$$

where $|\Psi^+\rangle$ is one of the Bell states $|\Psi^+\rangle = \frac{\sqrt{2}}{2}(|10\rangle + |01\rangle)$, the collective dephasing does not change the form of $r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4} I \otimes I$. So both the standard Werner state and the Werner-like state $r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4} I \otimes I$ belong to the decoherence-free subspace of the collective dephasing. They are robust states against the collective dephasing. Are all of the Werner-like states robust states against the collective dephasing? The answer is no. Now we consider another Werner-like state

$$\rho_W^{\pm} = r|\Phi^{\pm}\rangle\langle\Phi^{\pm}| + \frac{1-r}{4} I \otimes I, \tag{6}$$

where $|\Phi^{\pm}\rangle$ is the Bell states $|\Phi^{\pm}\rangle = \frac{\sqrt{2}}{2}(|11\rangle \pm |00\rangle)$. It is assumed that the initial state of master equation (3) is $\rho_W^{\pm}$. Then its time evolution density matrix can be expressed as

$$\rho_W^{\pm}(t) = re^{-2\gamma t}|\Phi^{\pm}\rangle\langle\Phi^{\pm}| + \frac{1-r}{4} I \otimes I + \frac{r}{2}(1 - e^{-2\gamma t})|11\rangle\langle 11| + \frac{r}{2}(1 - e^{-2\gamma t})|00\rangle\langle 00|. \tag{7}$$

In order to quantify the degree of entanglement, we adopt the concurrence C defined by Wooters [22]. The concurrence varies from $C = 0$ for an unentangled state to $C = 1$ for a maximally entangled state. For two qubits, in the "Standard" eigenbasis: $|1\rangle \equiv |11\rangle$, $|2\rangle \equiv |10\rangle$, $|3\rangle \equiv |01\rangle$, $|4\rangle \equiv |00\rangle$, the concurrence may be calculated explicitly from the following:

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{8}$$

where the $\lambda_i (i = 1,2,3,4)$ are the square roots of the eigenvalues *in decreasing order of magnitude* of the "spin-flipped" density matrix operator $R = \rho_s(\sigma^y \otimes \sigma^y)\rho_s^*(\sigma^y \otimes \sigma^y)$, where the asterisk indicates complex conjugation. The concurrence related to the density matrix $\rho_W^{\pm}(t)$ can be written as

$$C(t) = \max\left(0, \frac{r-1}{2} + re^{-2\gamma t}\right). \tag{9}$$

From equation (9), we can know that, different from the standard Werner state described by equation (1) and $r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4} I \otimes I$, the Werner-like state $r|\Phi^{\pm}\rangle\langle\Phi^{\pm}| + \frac{1-r}{4} I \otimes I$ rapidly loses its entanglement in the presence of collective dephasing. The threshold time t_c beyond which the entanglement of two qubits completely disappears can be obtained as

$$t_c = -\frac{1}{2\gamma}\ln\left[\frac{1-r}{2r}\right]. \tag{10}$$

Being similar to the results in reference [23], the Werner-like state described by equation (5) is completely disentangled in a finite time due to the collective dephasing if the initial parameter $r \neq 1$. It is also interesting to investigate how the collective dephasing affects the mixedness defined by $M = \frac{4}{3}(1 - \mathrm{Tr}\rho^2)$. The mixedness of the time evolution density matrix (6) can be calculated as

$$M(t) = 1 - \frac{r^2}{3} - \frac{2r^2}{3}e^{-4\gamma t}. \tag{11}$$

When $\gamma t \to \infty$, the final mixedness of the state in equation (6) equals to $1 - \frac{r^2}{3}$. It is shown that the state $r|\Phi^{\pm}\rangle\langle\Phi^{\pm}| + \frac{1-r}{4} I \otimes I$ loses its purity in the collective dephasing. The larger the parameter r, the smaller the final mixedness.

In the following, we attempt to discuss how the collective dephasing affects the Bell violation of the Werner-like states. Bell violation is not an entanglement measure. Those states violating the Bell inequality must be nonseparable. However, certain kinds of entangled states may not violate the Bell inequality. The most commonly discussed Bell inequality is the CHSH inequality [24,25]. The CHSH operator reads

$$\hat{B} = \vec{a}\cdot\vec{\sigma}\otimes(\vec{b}+\vec{b'})\cdot\vec{\sigma} + \vec{a'}\cdot\vec{\sigma}\otimes(\vec{b}-\vec{b'})\cdot\vec{\sigma}, \tag{12}$$

where $\vec{a}, \vec{a'}, \vec{b}, \vec{b'}$ are unit vectors. In the above notation, the Bell inequality reads

$$|\langle\hat{B}\rangle| \leq 2. \tag{13}$$

The maximal amount of Bell violation of a state ρ is given by [26]

$$\mathcal{B} = 2\sqrt{\lambda+\tilde{\lambda}}, \tag{14}$$

where λ and $\tilde{\lambda}$ are two largest eigenvalues of $T_\rho^\dagger T_\rho$. The matrix T_ρ is determined completely by the correlation functions being a 3×3 matrix whose elements are $(T_\rho)_{nm} = \mathrm{Tr}(\rho\sigma_n\otimes\sigma_m)$. Here, $\sigma_1\equiv\sigma_x$, $\sigma_2\equiv\sigma_y$, and $\sigma_3\equiv\sigma_z$ denote the usual Pauli matrices. We call the quantity $\mathcal{B}$ the maximal violation measure, which indicates the Bell violation when $\mathcal{B}>2$ and the maximal violation when $\mathcal{B}=2\sqrt{2}$. For the density matrix $\rho_W^{\pm}(t)$ in equation (6), $\lambda+\tilde{\lambda}$ can be written as follows

$$\lambda+\tilde{\lambda} = r^2\left(1+e^{-4\gamma t}\right). \tag{15}$$

The threshold time t_c^b beyond which the collective dephasing Werner-like state does not violate the Bell-CHSH inequality can be given by

$$t_c^b = -\frac{1}{4\gamma}\ln\left(\frac{1-r^2}{r^2}\right). \tag{16}$$

Equations (14, 15) show that, if $r\neq1$, i.e. the initial state is not a maximally entangled state, the dephasing Werner-like state in equation (7) rapidly loses its nonlocality in a finite time. The smaller the initial entanglement, the more rapidly the nonlocality completely disappears. In Figure 1, the threshold time t_c and t_c^b concerning the complete disentanglement and the disappearance of nonlocality of the state (7) respectively, have been plotted as the function of the parameter r. It implies that both the entanglement and nonlocality are completely destroyed in a finite time if the initial state is not pure. While the collective dephasing can not completely destroy nonlocality and entanglement of the pure Bell states $|\Phi^\pm\rangle$ in the finite time. According to two threshold times, we can classify the Werner-like state (6) by making use of the parameter r or the initial mixedness. If $r=0$, the state (6) reduces to the maximally mixed state. In the range of $0<r\leq\frac{1}{3}$, the states in equation (6) are separable. In the range of $\frac{1}{3}<r\leq\frac{\sqrt{2}}{2}$, they are entangled but not nonlocal. In the range of $\frac{\sqrt{2}}{2}<r<1$, they are inseparable and nonlocal, and both their nonlocality and entanglement can be completely destroyed by the collective dephasing. Recently, Yu and Eberly have shown a novel phenomenon that, under the influence of pure vacuum noise two entangled qubits become completely disentangled in a finite-time [23]. In a specific example they

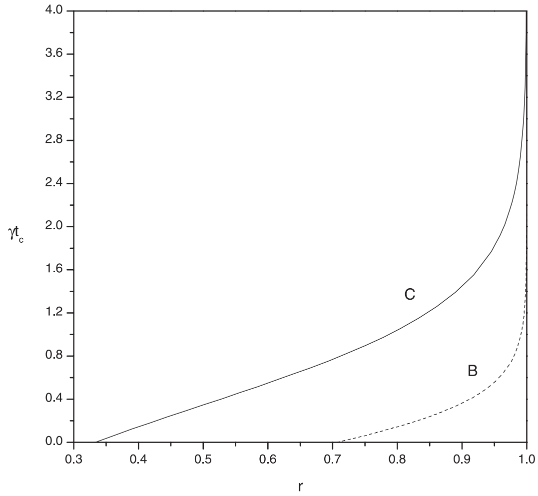

Fig. 1. The threshold time γt_c and γt_c^b of the concurrence and the maximal Bell violation respectively, are plotted as the function of the parameter r for the collective dephasing Werner-like states in equation (7); (solid line) γt_c in equation (10); (dash line) γt_c^b in equation (16). It is shown that both γt_c and γt_c^b increase with r.

have found the time to be given by $\ln(1+\frac{\sqrt{2}}{2})$ times the usual spontaneous lifetime. Here, we have obtained a similar result that in the presence of collective dephasing, two initial mixed entangled qubits become completely disentangled in a finite-time. It is conjectured that, in this case, the initial mixedness is an essential fact whether two entangled qubits become completely disentangled in a finite-time or not. In fact, previous work concerning the interaction between a initial mixed qubit and the thermal field in the presence of phase decoherence have revealed some analogical results [27].

It is also very interesting to investigate how an external driving field affects the complete disentanglement time in this situation. If the external driving fields are taken into account, the master equation (3) should be replaced by

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{2}\left[\Omega_1\hat{\sigma}_x^{(1)}+\Omega_2\hat{\sigma}_x^{(2)},\hat{\rho}\right] + \frac{\gamma}{2}(2\hat{J}_z\hat{\rho}\hat{J}_z - \hat{J}_z^2\hat{\rho} - \hat{\rho}\hat{J}_z^2), \tag{17}$$

where Ω_i $(i=1,2)$ are the intensity of the external driving field acted on the ith qubit. $\hat{\sigma}_x$ for each qubit are defined by $\hat{\sigma}_x = |1\rangle\langle0| + |0\rangle\langle1|$. In the case with $\Omega_1=\Omega_2=\Omega$, the standard Werner state is still decoupled from the above master equation, while other Werner-like states lose their entanglement. In order to know whether $r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4}I\otimes I$ and $r|\Phi^\pm\rangle\langle\Phi^\pm| + \frac{1-r}{4}I\otimes I$ can be completely disentangled in a finite time or not, it is sufficient to verify Bell states $|\Psi^+\rangle$ and $|\Phi^\pm\rangle$ are completely disentangled in their evolutions governed by equation (17). In Figure 2, the dynamical behaviors of concurrence of two qubits initially in Bell triplet states governed by equation (17) are displayed. It is shown that all of Bell triplet states are completely disentangled in a finite time, which imply all of the Werner-like states defined by equations (5) and (7) are finite-time disentangled in the joint action of the collective dephasing and external driving fields. From

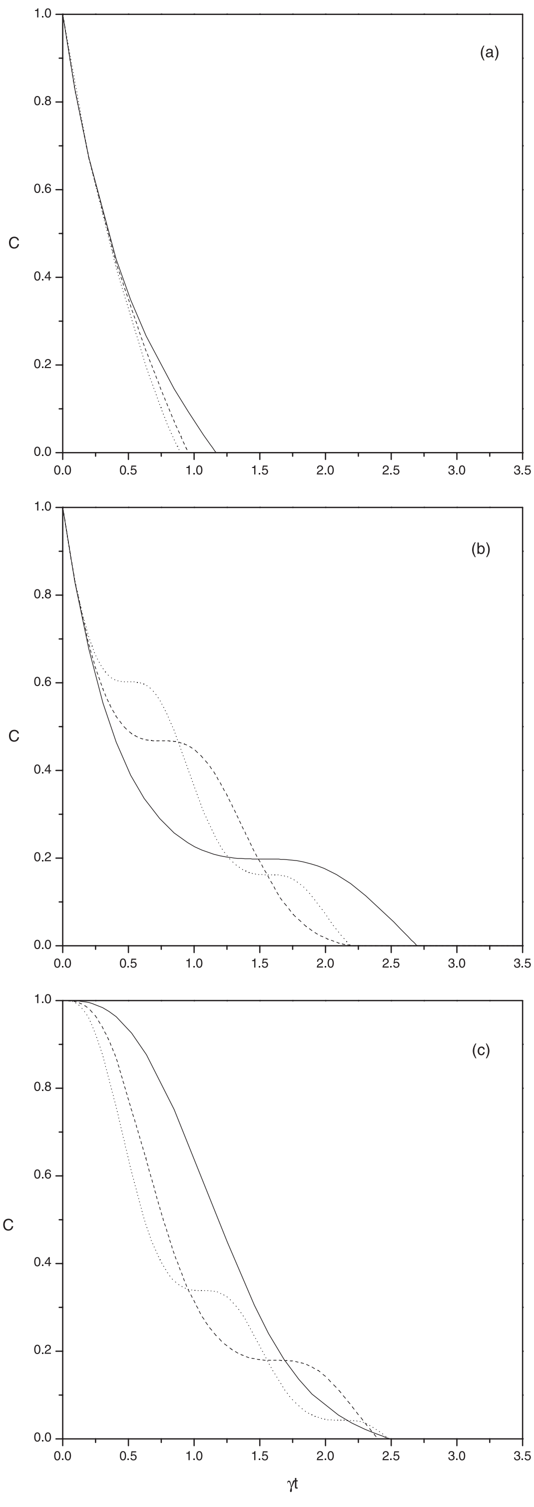

Fig. 2. The dynamical behaviors of concurrence of two qubits initially in Bell triplet states governed by equation (17) are displayed as the function of the dephasing time γt for different values of $\Omega_1 = \Omega_2 = \Omega$; (solid line) $\Omega = \gamma$, (dash line) $\Omega = 2\gamma$, (dot line) $\Omega = 3\gamma$. (a) Two qubits are initially in $|\Phi^-\rangle$; (b) two qubits are initially in $|\Phi^+\rangle$; (c) two qubits are initially in $|\Psi^+\rangle$. It is shown that all of Bell triplet states become separable in different finite times in the joint action of collective dephasing and driving fields. In the case without the driving fields, Bell triplet states can not become separable in finite time.

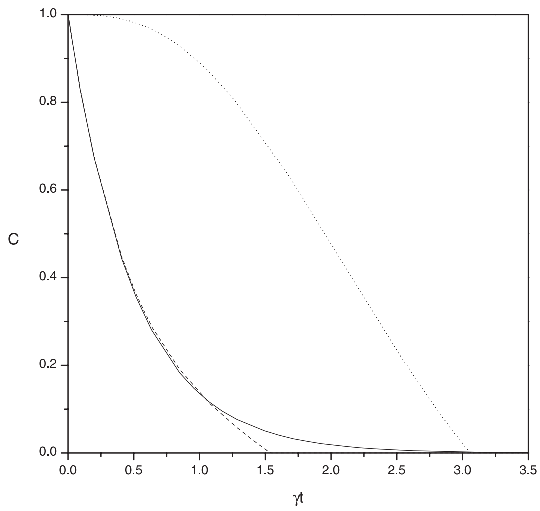

Fig. 3. The dynamical behaviors of concurrence of two qubits initially in four Bell states governed by the master equation (17) are displayed as the function of the dephasing time γt with $\Omega_1 = \gamma$ and $\Omega_2 = 0$. (Dash line) Two qubits are initially in $|\Phi^\pm\rangle$; (dot line) two qubits are initially in $|\Psi^\pm\rangle$; (solid line) for comparison, we display the asymptotic disentanglement of two qubits initially in $|\Phi^\pm\rangle$ in the presence of the pure collective dephasing without any external driving fields.

Figure 2, it can be observed that $|\Phi^-\rangle$ most rapidly loses its entanglement among all of Bell triplet states. In the case with $\Omega_1 \neq \Omega_2$, the Bell singlet state is no longer decoupled from the equation (17). In Figure 3, we show that four Bell states become complete disentanglement in the finite time in the situation with $\Omega_1 = \gamma$ and $\Omega_2 = 0$. The partially driving field acting on the qubit 1 destroys the symmetry of the pure collective dephasing and forces the Bell singlet state out of the decoherence-free subspace. Nevertheless, $|\Psi^\pm\rangle$ are more robust than $|\Phi^\pm\rangle$ in this case, and the threshold time corresponding to the complete disentanglement of two qubits initially in $|\Psi^\pm\rangle$ is about twice as large as the one corresponding to the complete disentanglement of two qubits initially in $|\Phi^\pm\rangle$.

In what follows, we briefly discuss a possible scheme to prevent the fragile entangled states from complete disentanglement under the action of collective dephasing. We assume that only one of two qubits is driven by a time-dependent external field. For simplicity, the time dependence of the external driving field is suggested to be the form of the unit step function. The dynamics of two qubits can be described by the following master equation

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{2}\left[\zeta_1(t)\hat{\sigma}_x^{(1)}, \hat{\rho}\right] + \frac{\gamma}{2}\left(2\hat{J}_z\hat{\rho}\hat{J}_z - \hat{J}_z^2\hat{\rho} - \hat{\rho}\hat{J}_z^2\right), \quad (18)$$

where $\zeta_1(t) = \zeta_1 \Theta(T - t)$ is the intensity of the time-dependent external driving field acted on the qubit 1, and $\Theta(x)$ is the unit step function and equals one for $x \geq 0$ and equals zero for $x < 0$. In previous experimental verification of the decoherence-free subspace [28], the external driving field and the collective dephasing are not simultaneously acted on the qubits. It is obvious that the local external driving field $\zeta_1(t)\hat{\sigma}_x^{(1)}$ can interconvert the Bell

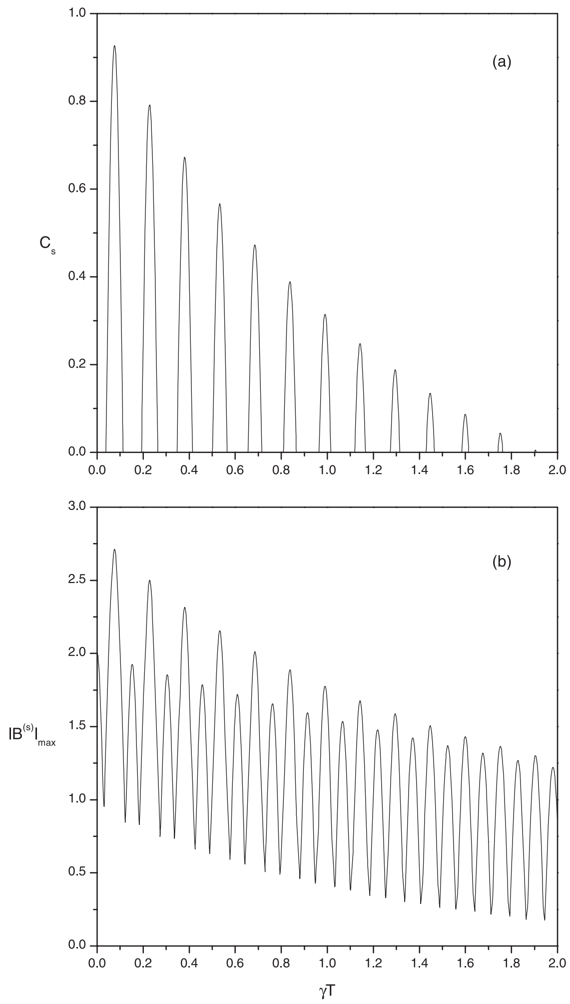

Fig. 4. The stationary state concurrence C_s (a) and the stationary state Bell violation $|B^{(s)}|_{max}$ (b) of two qubits initially in the Bell states $|\Phi^+\rangle$ governed by the master equation (17) are displayed as the function of the scaled action time γT with $\zeta_1/\gamma = 41.25$. In this case, when $\gamma T > 2$, there is not any stationary state entanglement between two qubits.

states without the simultaneous presence of collective dephasing. Nevertheless, it is desirable to determine the influence of the variation of T on the eventual stationary state entanglement when the driving field and the collective dephasing are simultaneously acted on the qubits in some realistic situations. Our numerical calculations show that the external field with an appropriate value of T can transform the initial fragile entangled state $|\Phi^+\rangle$ into a stationary entangled state even if the collective dephasing is always presence. In Figure 4, the stationary state concurrence C_s and the stationary state Bell violation $|B^{(s)}|_{max}$ of two qubits are plotted as the function of the parameter γT. It is shown that, if only the value of ζ_1/γ is large enough, one can maintain the entanglement and Bell violation of two qubits initially in a very fragile entangled state by making use of the local driving field with an appropriate action time T. We can see that the stationary state entanglement firstly increases with γT, and achieves a local maximal value, then decreases with γT. Similar behaviors are repeated again when the scaled action time γT is further enlarged. The stationary state Bell violation also oscillates with γT. The above calculations show one can effectively transform the fragile entangled state into a robust entangled state. This is meaningful and very important in many areas of quantum information processes.

Finally, we attempt to discuss the theoretical results about the fragile and robust Werner state in collective dephasing by comparing them with previous experimental results. In reference [28], Kwiat et al. have presented the experimental verification of decoherence-free subspace in the system of polarized photons. For investigating the influence of collective dephasing on the initial state, one can adopt the general fidelity $F(\rho_i, \rho_f) \equiv [\mathrm{Tr}(\sqrt{\rho_i}\rho_f\sqrt{\rho_i})^{1/2}]^2$ [29], where ρ_i is the initial state and ρ_f the final state. The fidelity between the initial Werner-like state $\rho_W^{\pm}$ in equation (6) and the corresponding dephasing state $\rho_W^{\pm}(t)$ in equation (7) can be calculated as

$$F_W = \frac{1}{16}[2(1-r) + \sqrt{(1+3r)(1+r+2re^{-2\gamma t})} + \sqrt{(1-r)(1+r(1-2e^{-2\gamma t}))}]^2, \quad (19)$$

which decreases with r and t. In this case, the minimal value of F_W is 0.5 at $r = 1$ and $t \to \infty$. The collective dephasing in the experiment of Kwiat et al. can be described by the following master equation which is equivalent to equation (3) under the local unitary transformation

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{\gamma}{2}(2\hat{J}_\theta\hat{\rho}\hat{J}_\theta - \hat{J}_\theta^2\hat{\rho} - \hat{\rho}\hat{J}_\theta^2), \quad (20)$$

where $\hat{J}_\theta$ are the collective spin operator defined by

$$\hat{J}_\theta = \sum_{i=1}^{2} \hat{\sigma}_\theta^{(i)}/2, \quad (21)$$

where $\hat{\sigma}_\theta$ for each qubit is defined by $\hat{\sigma}_\theta = \cos 2\theta\hat{\sigma}_z + \sin 2\theta\hat{\sigma}_x$. Throughout the following calculations, we choose $\theta = 17°$ for representing the realistic situation in reference [28]. The fidelity between the initial Werner-like states and corresponding stationary state of equation (20) for three different initial states has been calculated and the results are depicted in Figure 5. It was found that the fidelities decrease with r and the theoretical results excellently agree with the experimental data in reference [28]. By comparing two Table 1 in present paper and in reference [28], it may be interesting that the initial states in [28] look like the Werner or Werner-like states with very high purity. In Figure 6, the evolution of fidelities are investigated for three different initial states. It is shown that the fidelity decreases with dephasing time and eventually stays at a fixed value, which implies that these chosen initial states are out of the decoherence-free subspace of equation (20). It is shown that evolving fidelity of the initial state $0.99|\Phi^-\rangle\langle\Phi^-| + \frac{1}{400}I \otimes I$ is larger than the one of $0.99|\Phi^+\rangle\langle\Phi^+| + \frac{1}{400}I \otimes I$ in short time.

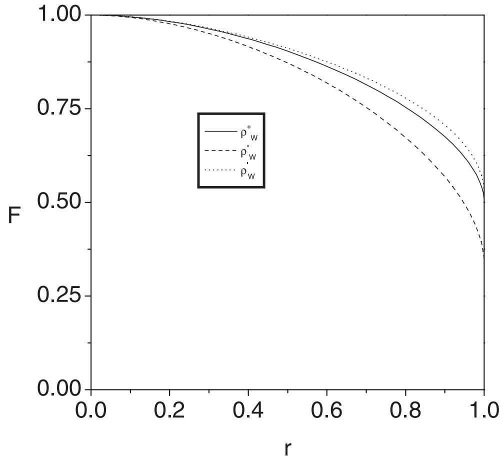

Fig. 5. The fidelity $F \equiv [\mathrm{Tr}(\sqrt{\rho_i}\rho_f\sqrt{\rho_i})^{1/2}]^2$ for three kinds of initial states and their corresponding complete dephasing states are plotted as the function of the parameter r. (Solid line) $\rho_i = \rho_W^+ = r|\Phi^+\rangle\langle\Phi^+| + \frac{1-r}{4}I \otimes I$; (dash line) $\rho_i = \rho_W^- = r|\Phi^-\rangle\langle\Phi^-| + \frac{1-r}{4}I \otimes I$; (dot line) $\rho_i = \rho_W' = r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4}I \otimes I$. It is shown that $F(\rho_W') > F(\rho_W^+) > F(\rho_W^-)$ for any non-zero values of r, where we have simplified the expression $F(\rho_i, \rho_f)$ as $F(\rho_i)$. This simplification has also been adopted in Table 1.

Table 1. The fidelities F_i $(i = 1, 2, 3, 4)$ of the initial states ρ_W^+ (in Eq. (6)), ρ_W^- (in Eq. (6)), ρ_W' (in Eq. (5)), ρ_W (in Eq. (1)) and their corresponding stationary states of equation (20) in order are listed for different values of r. $F_1 \equiv F(\rho_W^+)$; $F_2 \equiv F(\rho_W^-)$; $F_3 \equiv F(\rho_W')$; $F_4 \equiv F(\rho_W)$. We can see that present theoretical calculations based on equation (20) and the initial Werner-like state assumption well agree with the experimental data in Table 1 of reference [28].

r	1	0.99	0.98	0.97
F_1	0.500	0.552	0.575	0.592
F_2	0.334	0.403	0.434	0.457
F_3	0.521	0.582	0.606	0.625
F_4	1.000	1.000	1.000	1.000

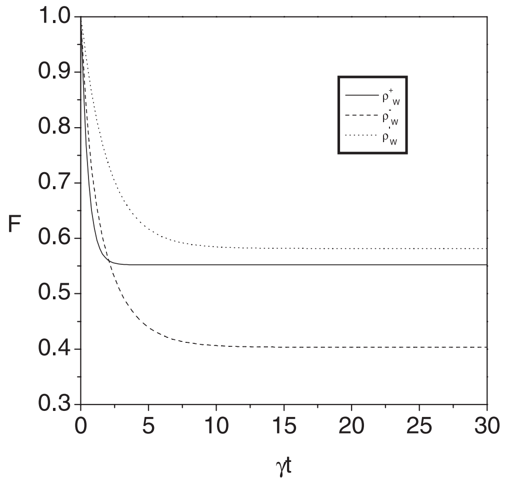

Fig. 6. The fidelities for three kinds of initial states and their corresponding evolving states governed by equation (20) are plotted as the function of γt. The initial Werner-like states with $r = 0.99$ are chosen for calculating all of three curves.

In summary, we investigate the concurrence and Bell violation of the standard Werner state or Werner-like states in the presence of collective dephasing. By making use of the analytical expressions of the concurrence and Bell violation obtained in the present paper, we find that the standard Werner state and certain kinds of Werner-like states are robust against the collective dephasing, and some kinds of Werner-like states is fragile and becomes completely disentangled in a finite-time. The threshold time of complete disentanglement of the fragile Werner-like states is analyzed. We conjecture that the initial mixedness is an important fact to determine whether two entangled qubits become completely disentangled in a finite-time or not in the pure collective dephasing. Moreover, the threshold time concerning the complete disappearance of the Bell violation of some kinds of fragile Werner-like states is also obtained. Furthermore, we investigate how an external driving field affects the completely disentanglement time and clarify that the pure maximally entangled states of two qubits can become separable in the finite time in this situation. Since the standard Werner state or Werner-like states play a special role in some quantum information processes, such as quantum teleportation, our results may have potential applications in quantum teleportation [30,31] or other remote quantum information processes.

We also discuss the possible way to transform the fragile entangled state into the robust entangled state in the collective dephasing environment. It is shown that a local external driving field with an appropriate finite action time can effectively maintain both the entanglement and Bell violation of two qubits even if the collective dephasing is presence from beginning to end.

Recently, the quantum information processes in the presence of the collective dephasing have intrigued much attention [13,14,32–34]. Khodjasteh and Lidar have investigated the universal fault-tolerant quantum computation in the presence of spontaneous emission and collective dephasing [32]. Hill and Goan have studied the effect of dephasing on proposed quantum gates for the solid-state Kane quantum computing architecture [33]. In the future work, it may be very interesting to apply the present results to discuss the influence of collective dephasing on gate fidelity of remote quantum computation.

Finally, we also compare the theoretical results about the fidelity of the initial Werner-like state in the presence of collective dephasing with recent experimental data of Kwiat et al. [28]. It is shown that they are consistent with each other.

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